5. I. S. Zhitomirskii, L. A. Borisenko, L. A. Ishchenko, and V. A. Pestryakov, "Graphical problems in nonstationary heat transfer and methods for their solution," Heat Transfer, V, Vol. VII. Heat Transfer by Radiation and Complex Heat Transfer [in Russian], ITMO, Akad. Nauk BSSR, Minsk (1976).

# RECONSTRUCTION OF LOCAL EQUILIBRIUM TEMPERATURE FIELDS IN AN EMISSIVE MEDIUM

V. V. Pikalov and N. G. Preobrazhenskii UDC 533.9

A spectral method for determination of local temperatures in an emitting volume is discussed. The problem of reconstruction of emissivity in the case of a medium of arbitrary configuration is solved by regularization.

In a number of thermophysical problems, it is often necessary to determine temperature fields within an emitting volume of plasma or of a high-temperature gas flow. The use of the methods of emission and absorption spectroscopy makes it possible to obtain the necessary pyrometric information without introducing perturbations in the test medium. The procedure for finding the temperature T(x, y) after determination of the emissivity $\varepsilon(x, y)$ and absorptivity $\varkappa(x, y)$ has been developed satisfactorily [1, 2], but generally the search for these functions is a complex inverse problem. Actually, it is necessary to determine the coefficients of the radiation-transport equation from values I(S) of the solution of this equation measured on the boundary of the volume. The main results in this problem were obtained with reference to the particular case of axial symmetry where the problem becomes one-dimensional. If the absorptivity is negligibly small, (optically thin layer), the problem reduces to a solution of the Abelian integral equation [1]

$$I(x) = 2\int_x^R \frac{\varepsilon(r)\,rdr}{\sqrt{r^2 - x^2}}, \tag{1}$$

where R is the radius of the emitting volume. However, cases with elliptical symmetry in $\varepsilon(x, y)$ can also be reduced to such an equation. Let the orientation of an ellipse with semiaxes $a$ and b be characterized by the parameter t:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{t^2}{a^2}, \quad b < a, \tag{2}$$

$$x \in [-a, a], \quad y \in [-b, b], \quad t \in [0, a].$$

Making measurements along the y axis, we obtain

$$I(x) = 2\int_0^{b\sqrt{1-\frac{x^2}{a^2}}} \varepsilon(x, y)\,dy = 2\,\frac{b}{a}\int_x^a \frac{\varepsilon(t)\,tdt}{\sqrt{t^2 - x^2}}, \tag{3}$$

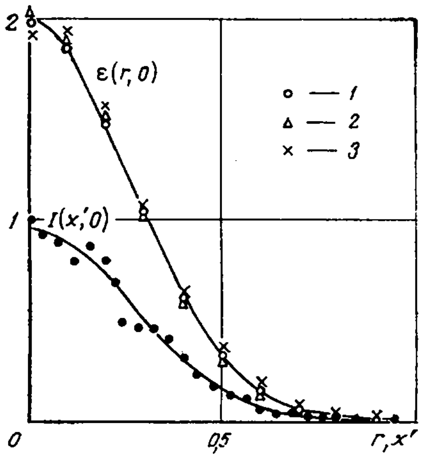
i.e., once again an Abelian equation but with respect to the isolines of an ellipse rather than a circle as in Eq. (1). A deficiency of such a treatment of elliptical symmetry is the need for preliminary experimental determination of the orientation of the test elliptical object in the laboratory coordinate system.

A large amount of work was devoted to solution of the Abelian equation by various methods including the use of regularization of one kind or another [3-6]. A comparison was made [7] of a number of methods with respect to the intensification of the experimental errors in them.

In the general case, the lack of symmetry in the problem is expressed in the form of an integral equation of the first kind:

---

Institute of Theoretical and Applied Mechanics, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1042-1046, December, 1977. Original article submitted April 5, 1977.



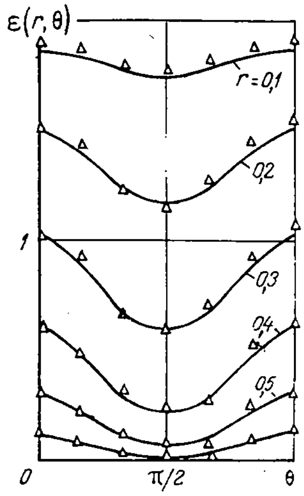




Fig. 1 Fig. 2

Fig. 1. Model functions $\varepsilon(r, 0)$, $I(x', 0)$, and results of the reconstruction of $\varepsilon(r, 0)$ ($a = 2.7$, $b = 3.7$, $d = 0$, $B = 1$, $\alpha = 2.5$). Coefficients of variation: 1) 5; 2) 10; 3) 15%.

Fig. 2. Function $\varepsilon(r, \theta)$ and results of its reconstruction by regularization (parameters same as in Fig. 1).

$$I(x', \xi) = \int_{-\infty}^{\infty} \varepsilon(x, y)\, dy', \tag{4}$$

where the laboratory reference frame $(x', y')$ is rotated by an angle $\xi$ with respect to the coordinate system $(x, y)$ fixed in the object.

To solve this equation, expansions in terms of various systems of orthogonal polynomials [8-11] or the Radon transformation [12-13] were used but subsequent regularization was not performed.

The present work considers an algorithm for regularization of the solution of Eq. (4) based on an expansion of the input data vector in a generalized Fourier series in terms of a special system of orthogonal polynomials [14] which are invariant with respect to rotation by limitation of summation in accordance with the discrepancy principle [15].

Transforming to the polar coordinate system $x = r\cos\theta$, $y = r\sin\theta$, we write the function sought in the form

$$\varepsilon(r, \theta) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{\infty}\sum_{k=0}^{\infty} w_m (-1)^k \frac{k!}{(m+2k)!} \exp(-\alpha^2 r^2)$$
$$\times [B^m_{m+2k}(\alpha)\cos m\theta + D^m_{m+2k}(\alpha)\sin m\theta](\alpha r)^m L^m_k(\alpha^2 r^2). \tag{5}$$

Here, $w_0 = 0.5$, $w_m = 1$ $(m \neq 0)$; $L^m_k(\alpha^2 r^2)$ are generalized Laguerre polynomials; $\alpha$ is a parameter influencing the rate of convergence of the series, and

$$B^m_{m+2k}(\alpha) = \int_0^{\pi} \cos m\xi\, d\xi \int_{-\infty}^{\infty} [I(x', \xi) H_{m+2k}(\alpha x') + I(x', \pi - \xi) H_{m+2k}(-\alpha x')]\, dx',$$
$$D^m_{m+2k}(\alpha) = \int_0^{\pi} \sin m\xi\, d\xi \int_{-\infty}^{\infty} [I(x', \xi) H_{m+2k}(\alpha x') - I(x', \pi - \xi) H_{m+2k}(-\alpha x')]\, dx', \tag{6}$$

where $H_{m+2k}(\alpha x')$ are Hermite polynomials.

In the following, we consider cases where the function $I(x', \xi)$ is symmetric with respect to $\xi = 0$ (symmetry plane) so that the last expressions simplify to

$$D^m_{m+2k}(\alpha) = 0;$$

$$B_{m+2k}^{m}(\alpha)=2\int_{0}^{\pi}\cos m\xi d\xi\int_{-\infty}^{\infty}I(x',\ \xi)H_{m+2k}(\alpha x')\,dx'; \tag{7}$$

$$\varepsilon(r,\ \theta)=\left(\frac{\alpha}{\pi}\right)^{2}\exp(-\alpha^{2}r^{2})\sum_{m=0}^{\infty}w_{m}(\alpha r)^{m}\cos m\theta\sum_{k=0}^{\infty}\frac{(-1)^{k}k!}{(m+2k)!}B_{m+2k}^{m}(\alpha)\,L_{k}^{m}(\alpha^{2}r^{2}). \tag{8}$$

The coefficients $B_{m+2k}^{m}(\alpha)$ in the Fourier expansion are determined with included error because of the experimental nature of the function $I(x',\ \xi)$ and hence the problem of summing the series (8) is an improper problem [16].

One of the stable methods for summation of the Fourier series is limitation of the number of terms in the series compatible with experimental error. Substituting the series (8) in Eq. (4) and carrying out simple transformations, we obtain as an expression for the discrepancy

$$\Delta(x',\ \xi)=I(x',\ \xi)-\pi^{-3/2}\exp(-\alpha^{2}x'^{2})\left[0.5\sum_{k=0}^{K}z_{2k}^{0}(\alpha x')\,C_{2k}^{0}(\alpha)\right.$$
$$\left.+\sum_{m=1}^{M}m^{-1}\cos m\xi\sum_{k=0}^{K}z_{m+2k}^{m}(\alpha x')\,C_{m+2k}^{m}(\alpha)\right], \tag{9}$$

where

$$z_{m+2k}^{m}(\alpha x')=2^{-m-2k}[k!(m+k)!]^{-1/2}H_{m+2k}(\alpha x'),$$
$$C_{m+2k}^{m}(\alpha)=[(m+2k)!]^{-1}m\alpha[k!(m+k)!]^{1/2}B_{m+2k}^{m}(\alpha). \tag{10}$$

The algorithm for regularization was constructed on the basis of a search for values of K and M such that $\|\Delta(x',\ \xi)\|_{l_2}=\delta^{2}$, where $\delta$ is the norm of the error of a given function $I(x',\ \xi)$. The algorithm given was applied to the model functions

$$\varepsilon(r,\ \theta)=2B\exp[-r^{2}(a^{2}\sin^{2}\theta+b^{2}\cos^{2}\theta)-b^{2}d^{2}]\,\mathrm{ch}(2b^{2}rd\cos\theta),$$
$$I(x',\ \xi)=\frac{2B\sqrt{\pi}}{\sqrt{a^{2}\cos^{2}\theta+b^{2}\sin^{2}\theta}}\exp\left(-b^{2}d^{2}-\frac{a^{2}b^{2}x'^{2}-b^{4}d^{2}\sin^{2}\theta}{a^{2}\cos^{2}\theta+b^{2}\sin^{2}\theta}\right)$$
$$\times\,\mathrm{ch}\left[-2x'db^{2}\cos\xi\left(1-\frac{(a^{2}-b^{2})\sin^{2}\xi}{a^{2}\cos^{2}\xi+b^{2}\sin^{2}\xi}\right)\right], \tag{11}$$

which satisfy Eq. (4) exactly.

Figure 1 shows the general form of the functions $\varepsilon(r,\ 0)$ and $I(x',\ 0)$, and Fig. 2 presents solutions obtained with the scheme discussed above. To reduce the consumption of machine time, a discrepancy criterion in the form $\|\Delta(x',\ 0)\|=\delta_{2}^{1}$ was used. Experimental errors were simulated by a normal random process with coefficients of variation of 5, 10, and 15%.

As shown by the figures, the algorithm developed shows little intensification of experimental "noise" and can be recommended for analysis of spectral measurements in a thermophysical experiment.

## NOTATION

$I(x',\ \xi)$, recorded radiation intensity; $\varepsilon(r,\ \theta)$, local emissivity; t, parameter; $a$, b, major and minor semiaxes of ellipse; $B_{m+2k}^{m}(\alpha)$, $D_{m+2k}^{m}(\alpha)$, coefficients of generalized Fourier series; $\Delta(x',\ \xi)$, discrepancy function; B, constant coefficient; $\alpha$, scale factor; $z_{m+2k}^{m}(\alpha x')$, special polynomials; K, limit of summation of series over k; M, limit of summation of series over m.

## LITERATURE CITED


1. H. R. Griem, Plasma Spectroscopy, McGraw-Hill (1964).
2. R. MacWhirter, in: Plasma Diagnostic Techniques (edited by R. Huddlestone and S. Leonard), Academic Press (1965).
3. G. Minerbo and M. Levy, SIAM J. Numer. Anal., 6, No. 4 (1969).
4. M. I. Pergament, in: Physics and Application of Plasma Accelerators [in Russian], Nauka i Tekhnika, Minsk (1974).

5. V. V. Pikalov and N. G. Preobrazhenskii, Fiz. Goreniya i Vzryva, 10, No. 6 (1974).
6. N. A. Magnitskii and S. A. Magnitskii, Preprint No. 118, Physics Institute, Academy of Sciences of the USSR, Moscow (1976).
7. V. V. Pickalov, N. G. Preobrazhenskii, and G. I. Smirnova, Proceedings of the Eighth International Symposium on Discharging Electrons in an Insulated Vacuum, Novosibirsk (1976).
8. H. N. Olsen, C. D. Maldonado, and G. D. Duckworth, J. Quant. Spectrosc. Radiat. Transfer, 8, No. 7 (1968).
9. D. Sweeney and C. Vest, Appl. Opt., 12, No. 11 (1973).
10. B. K. Vainshtein, Usp. Fiz. Nauk, 109, No. 3 (1973).
11. R. Bracewell and A. Riddle, Astrophys. J., 150, No. 2, 1 (1967).
12. I. N. Shtein, Radiotekh. Radioélektron., 17, No. 11 (1972).
13. Yu. P. Presnyakov, Opt. Spektrosk., 40, No. 1 (1976).
14. H. N. Olsen, C. D. Maldonado, C. D. Duckworth, and A. P. Caron, Contract AF 33(615)-1105, ARL 66-0016, California (1966).
15. V. A. Morozov, Zh. Vychisl. Mat. Mat. Fiz., 8, No. 2 (1968).
16. A. N. Tikhonov and V. Ya. Arsenin, Methods for Solution of Improper Problems [in Russian], Nauka, Moscow (1974).

# THE TEMPERATURE DEPENDENCE OF THE THERMAL CONSTANTS OF COMPOSITE POLYMER MATERIALS

V. I. Strakhov, S. I. Leonova,
and A. N. Garashchenko

UDC 536.21

We developed a method for determination of the thermal-conductivity temperature dependence of organic and fiberglass plastics at temperatures up to 1000°C from thermocouple measurements by solving the inverse heat-conduction problem.

In the region of temperatures exceeding the minimum temperature of thermal decomposition of composite polymer materials, the macrostructure and the chemical composition of the material change and the thermal effects of the physicochemical transformations appear. These factors depend crucially on the rate and conditions of heating and as a result, the traditional methods of measuring the thermal constants [1] are largely inapplicable since these are based on the solution of the heat equation without taking into account the features mentioned above. The determination of the thermal constants in this temperature region is made possible using the temperature measurements in heating conditions close to those occurring in real situations by the method of the inverse heat-conduction problem (IHCP).

The mathematical model describing the heat propagation in the composite polymer materials at high temperatures should describe all relevant features of the process, and at the same time be sufficiently simple from the point of view of practical applications. These requirements are satisfied by the heat equation written in the form

$$F\frac{\partial t}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial t}{\partial x}\right)+G\frac{\partial t}{\partial x};$$
$$F=(1-f)c_2\rho_2+\rho_0(1-K_{\mathrm{mc}})Q\frac{\partial \chi}{\partial t};\quad G=c_1\int\limits_{x_{\mathrm{bd}}}^{x}\frac{\partial \chi}{\partial \tau}\,dx. \tag{1}$$

The majority of the physical parameters appearing in the heat equation (1) ($\chi$, $K_{\mathrm{mc}}$, $f$, $Q$, $\rho_0$, $\rho_2$, $c_1$) can be determined by existing methods. To determine the specific heat $c_2$ and the thermal conductivity $\lambda$ it is necessary to use IHCP.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1047-1051, December, 1977. Original article submitted April 5, 1977.

 0022-0841/77/3306-1454$07.50 